Multiplication and Division

**Words to know:**
- multiplication
- product
- division
- quotient
- dividend
- divisor

**Multiplication** is repeated addition. The answer to a multiplication problem is called the *product*. Let’s take a look!

**Example:** You have spent the day at the beach and collected many small stones. Now you want to count them. You decide to make groups of five stones each. You find that you have nine groups of five stones. How many stones is that all together?

\[ 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 45 \]

You have forty-five stones all together. Instead of adding 5 nine times, you could have used multiplication.
✓ Multiplication is the same as adding a number many times, or repeated addition. 9 x 5 is the same as adding 5 to itself nine times. There are 9 groups of 5. The symbol we use for multiplication is “×”.

✓ The answer to a multiplication problem is called the product. For example, 3 x 2 = 6. 4 times 3 is 12. The product of 7 and 5 is 35.

Many rules that apply to addition also apply to multiplication. With addition, you can add numbers in any order and still get the same answer.

Example: 3 + 2 = 5 and 2 + 3 = 5.

The same rule is true of multiplication:

Example: 9 x 5 = 45 and 5 x 9 = 45.

Nine groups of five is the same as five groups of nine.

There is a set of steps to follow when you want to multiply larger numbers.

Example: Esmeralda is making cookies at the bakery she works for. She mixes enough dough to fill 16 cookie trays with 12 cookies each. How many cookies will Esmeralda bake all together?
Solution: She is filling 16 trays each with 12 cookies. That means she will have 16 groups of 12. In other words, you must find the product of 16 and 12, or 16 x 12. Start by arranging the numbers vertically, as you did before with addition and subtraction.

Step 1: Multiply the two and the six. $6 \times 2 = 12$. Put the two from the 12 below the digits column, and carry the one to the tens place.

Step 2: Multiply the two and the one. $2 \times 1 = 2$. Add this to the 1 that you carried from the first step. $2 + 1 = 3$. Write the three below, in the

Step 3: Cross out the 2 and the 1 that you carried. Put a zero beneath the 32 that lines up with the 2.
Step 4: Multiply the 1 and 6.  
\[ 1 \times 6 = 6. \]
Write the 6 under the 3.

Step 5: Multiply the 1 and the other 1.  \[ 1 \times 1 = 1. \]
Don’t do anything with the remainder you crossed out. Write the 1 next to the 6.

Step 6: Finally, add the two products.  
\[ 32 + 160 = 192 \]

Esmeralda made 192 cookies.
Example: Find the product of $15 \times 13$.

Solution: Here we will illustrate each step in a more condensed way. Study each step from left to right, and notice the changes.

\[
\begin{array}{cccccc}
15 & 15 & 15 & 15 & 15 \\
\times 13 & \times 13 & \times 13 & \times 13 & \times 13 \\
45 & 45 & 45 & 45 & 45 \\
0 & 150 & + 150 & 195
\end{array}
\]

Your multiplication work should look like the column on the far right.

Now you try!

1. Find the products.

   a. $3 \times 2 =$  
   b. $9 \times 7 =$  
   c. $5 \times 3 =$ 

   d. $7 \times 4 =$  
   e. $3 \times 9 =$  
   f. $5 \times 11 =$ 

   g. $2 \times 4 =$  
   h. $12 \times 5 =$  
   i. $8 \times 8 =$
2. Find the products.  
   a. \[27 \times 23\]  
   b. \[13 \times 13\]

3. Isabel wants to know how much gas she buys in a year. Her car holds 12 gallons of gas. If she fills up her car 24 times all year, how many gallons of gas has she bought that year?

**Division**

You collected forty-five stones from your day at the beach. Your friend, Alejandro, brings you thirty more. You want to count them in groups of five. How many groups of five stones can you make with 30 stones?

To find out, you must use division. You must find the **quotient** of 30 ÷ 5.

**Division** is the process of separating something into smaller, equally sized groups. It is **repeated subtraction**.

- The **quotient** is the answer to a division problem.
- The **dividend** is the number you are separating into groups (30 above). It is the number you subtract from.
- The **divisor** (5 above) is the number that does the dividing. It can be the size of each group after division. Or, it can be the number of groups a number is divided into.
Look at this example of a division problem: $6 \div 2 = 3$. Six is the dividend.

Three is the quotient. Two is the divisor. It means that there are three groups of two in the number six. It also means that six can be divided into two groups of three.

Now, back to your problem of finding out how many groups of five 30 can be divided into. Use integer chips to represent the stones. Let one positive integer chip represent one stone.

Make groups of 5 positive chips each.

You can make six groups. Six groups of five stones is thirty stones. This sounds like multiplication, doesn't it? Specifically, $6 \times 5 = 30$. You could have answered this division question by turning it into a multiplication problem.

$30 \div 5 = \_\_\_\_\_\_\_\_$ is the same as asking $5 \times \_\_\_\_\_\_\_\_\_\_ \_\_\_ = 30$.  

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**Example:** Find the quotient of 28 and 4.

**Solution:** The first thing to understand is that *quotient* means to divide. You must find the answer to $28 \div 4$. You know you can change this into a multiplication problem.

$4 \times \_\_ = 28$

Now the question is easier to answer.

If you do not know this multiplication fact in your head, write out something like this:

\[
\begin{align*}
4 \times 1 &= 4 \\
4 \times 2 &= 8 \\
4 \times 3 &= 12 \\
4 \times 4 &= 16 \\
4 \times 5 &= 20 \\
4 \times 6 &= 24 \\
\textbf{4} \times 7 &= 28
\end{align*}
\]

You see that $4 \times 7 = 28$. The answer to the division problem, or quotient, is 7.

Just as you finish solving the division problem, Alejandro brings you more stones. This time, there are 24 stones to divide into groups of 5. With integer chips, $24 \div 5$ looks like the example on the next page:
There are four perfect groups of 5 stones and 4 stones left over.

In math, this leftover four is called a remainder.

- The remainder is the amount left over after division. To show a remainder, put an upper case “R” after the number of even divisions. Write the remainder number directly after the “R”.
- For example, the problem above would be written as: $24 \div 5 = 4 \ R 4$

FACT

The remainder should always be smaller than the divisor. If it is larger than the divisor, you need to keep dividing until the remainder is less than the divisor.
You learned that multiplication is really repeated addition, or adding many times.
Division can be shown as repeated subtraction. You can think of the stones problem, 24 ÷ 5, like this:

“How many times can I subtract 5 from 24 without making it negative?”

You can subtract 5 from 24 four times, and then have 4 left over. That means your answer is 4 R4.

Thinking of division as repeated subtraction helps make a rule for dividing larger numbers.
Example: Find $74 \div 3$.

Solution: In the number 74, 7 is in the tens place, and 4 is in the ones place. You can also say that there are 7 tens and 4 ones. Visually, it looks like this:

![Diagram of 74 divided into three groups]

You need to divide 74 into three groups.

First, group the tens.

![Diagram of tens grouped into three sets]

Two tens evenly fit into each group. One ten and four ones are left over. You cannot divide 10 ones equally among the 3 groups. You must keep it as 10 ones, and add the remaining 4. Now, you have 14 ones to divide into the three groups.
You can see that each group contains 2 tens and 4 ones. There are 2 ones remaining. The 2 ones cannot be divided evenly into the three groups. So, the answer is 24 $R2$. 
This method can be shown with numbers too.

Instead of writing $74 \div 3$, you can write $3 \overline{)74}$.

It means the exact same thing. Then, just as you did with the blocks, divide the tens place by three.

\[
\begin{array}{cccc}
2 & \downarrow \\
3)74 & \hline \\
6 & \\
1 & \\
\end{array}
\]

**Step 1:** 7 tens divide into 3 groups of 2 tens

**Step 2:** 3 groups of 2 tens is 6 tens

**Step 3:** There is still one ten left to divide, which does not divide evenly into three groups.

At this point in the visual model, you broke the group of ten into 10 ones, and combined it with the 4 ones. This is shown as follows.

\[
\begin{array}{cccc}
2 & \downarrow & \downarrow \\
3)74 & \hline & \hline \\
6 & \downarrow & \\
14 & \\
\end{array}
\]

**Step 4:** Combine the 4 ones with the 1 ten to get 14 ones. Now we must divide 14 ones into 3 groups.

**Step 5:** 14 ones divide into 3 groups of 4.

\[
\begin{array}{cccc}
2 & \downarrow \\
3)74 & \hline \\
6 & \\
14 & \\
\end{array}
\]

**Step 6:** 3 groups of 4 ones is 12 ones.

**Step 7:** Notice there are still 2 ones remaining. This is the remainder.
The final solution, with all the work, will look like this,

\[
\begin{array}{c}
24 \\
\hline
3 \overline{74} \\
-6 \\
\hline
14 \\
-12 \\
\hline
2 \\
\end{array}
\]

Step 8: Write the remainder.

Now you try!

4. Find the quotients by rewriting the division problems as multiplication problems.
   (For example, to find 10 ÷ 2 = ___, you would write 2 x ___ = 10.
    Then fill in 2 x 5 = 10.)

   a. 12 ÷ 2 = ___
   b. 16 ÷ 4 = ___
   c. 50 ÷ 25 = ___

   d. 24 ÷ 8 = ___
   e. 35 ÷ 7 = ___
   f. 18 ÷ 2 = ___

   g. 100 ÷ 4 = ___
   h. 20 ÷ 5 = ___
   i. 36 ÷ 12 = ___
Use the step by step method to find the quotients. There may be remainders.

5. $2\overline{28}$
6. $2\overline{428}$
7. $4\overline{27}$

8. $3\overline{32}$
9. $5\overline{223}$
10. $6\overline{1000}$

11. A school is divided into grades nine, ten, eleven, and twelve. Each grade has the same number of students. If there are 1,424 students in the school, how many students are in grade ten?
Multiplying and Dividing with Negative Numbers

When you multiply and divide negative numbers, you must pay attention to their signs (+ or −).

Example: The Iditarod (I-did-a-rod) is a dog-sled race run in Alaska every year. Temperatures during the race can go far below zero. One year, the temperature at one checkpoint in the race was −35°. From there, the racers traveled up a mountain. At the next checkpoint, the temperature was twice as cold. What is the new temperature at the second checkpoint?

Solution: In this problem, the picture above and the information about the Iditarod are not necessary. Take only what is important. You are told that the first temperature is −35°. The second temperature is twice as cold as −35°. In math terms, you can write this temperature change as

\[-35 \times 2\]

This is the first time you have seen negative numbers with multiplication. Think about the meaning of the statement, −35 x 2. It means “two groups of negative 35.” You can show that with integer chips. Think back or look back to Lesson 1.

Think Back

You can multiply numbers in any order you want. It still means the same thing!

3 x 2 = 2 x 3 = 6
From before, here is a positive chip \( + \) = +1 and a negative chip \( - \) = -1.

Now we must show two groups of \(-35\). Here they are.

If you count, there are 70 negative integer chips. They represent \(-70\). That means that \(-35 \times 2 = -70\). The temperature at the second checkpoint was \(-70\).

Using integer chips to multiply and divide negative numbers can become difficult. There are rules that make the process easier.
Rule to multiply or divide two integers:
1. Ignore the signs (+ or −) in front of the integers.
2. Multiply or divide as if they are both positive.
3. Write down the product or quotient.
4. Now look back at the signs of the original two numbers.
   a. If they are the same, your answer will be positive (+).
   b. If they are different, your answer will be negative (−).

Example: Simplify $12 \div -3$

Solution: First set this up as a division problem and solve it ignoring the signs.

\[
\begin{array}{c|c}
3 & 12 \\
\hline
3 & 12 \\
\hline
0 & 4
\end{array}
\]

You end up with 4 and no remainder. Now, look back to your original numbers, 12 and −3. The signs here are positive (+) and negative (−). These signs are different, so we know the answer will be negative.

Therefore, $12 \div -3 = -4$.

Note that this answer is also the correct solution to $-12 \div 3$. 
Example: Find the product. \(-12 \times -9\)

**Solution:** First multiply as you would with positive numbers.

\[
\begin{array}{c@{}c@{}c@{}c@{}c@{}c@{}c@{}c@{}c@{}c}
& & & & 1 & 2 & & & & \\
\times & & & & 9 & & & & & \\
\hline
& & & & & & 1 & 0 & 8 & \\
\end{array}
\]

Now, look at the signs of the original two numbers, \(-12\) and \(-9\).
Both signs are the same, so the answer will be positive.

Therefore, \(-12 \times -9 = 108\).

**Now you try!**

12. Find the products.

   a. \(-2 \times -3\)   b. \(-4 \times 2\)   c. \(6 \times 5\)   d. \(8 \times -4\)

   ___   ___   ___   ___

   e. \(-9 \times -7\)   f. \(-6 \times 4\)   g. \(12 \times -11\)   h. \(8 \times 7\)

   ___   ___   ___   ___

13. Find the quotients.

   a. \(-8 \div -2\)   b. \(14 \div -7\)   c. \(-20 \div 10\)   d. \(18 \div 9\)

   ___   ___   ___   ___

   e. \(-25 \div 5\)   f. \(24 \div -6\)   g. \(-100 \div -10\)   h. \(-2 \div 1\)

   ___   ___   ___   ___
Notes:

End of Lesson 3