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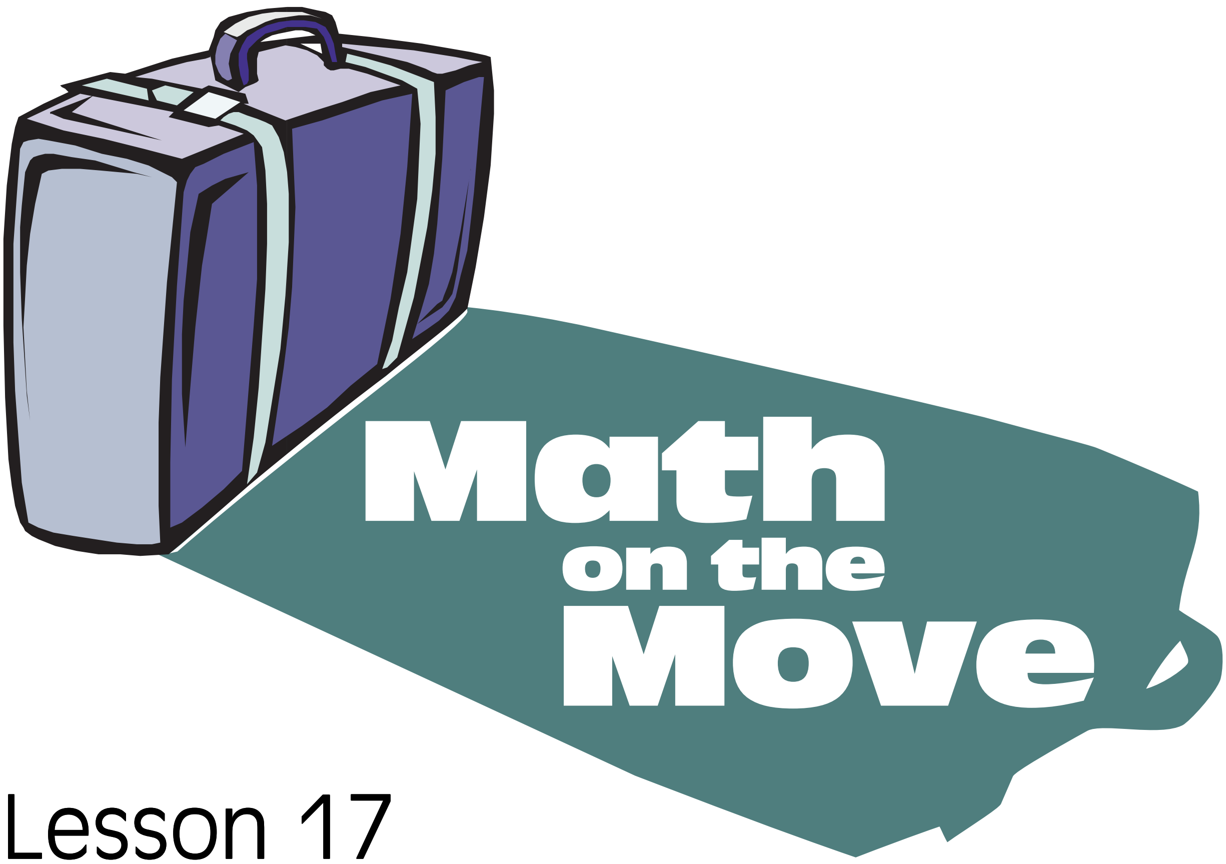

Lesson 17

Introduction to Geometry

Objectives

- Understand the definitions of points, lines, rays, line segments
- Classify angles and certain relationships between lines

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The word *geometry* comes from Greek. It literally means "Earth-measure." It is the branch of math that studies the interesting relationships caused by the size and shape of objects. To study geometry, we need to first know some basic terms.

- A **point** is a location in space. It is shown with a dot. •
- A **line** goes straight through two points in opposite directions and never ends.

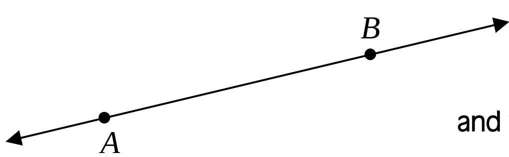
  - The arrows show that the line has no end. It goes on forever in both directions. The notation we use to represent this line is

$$\overleftrightarrow{AB} \text{ or } \overleftrightarrow{BA}.$$

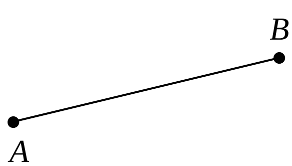
- A **ray** goes through two points, has one endpoint, and extends without end in one direction

  - The ray above shows an endpoint at *A*. It continues forever in one direction through the point *B*. The notation for the ray is $\overrightarrow{AB}$.

Imagine we have a line, say $\overleftrightarrow{AB}$, drawn below.



and we cut the line at points *A* and *B*.



This is called a **line segment**.

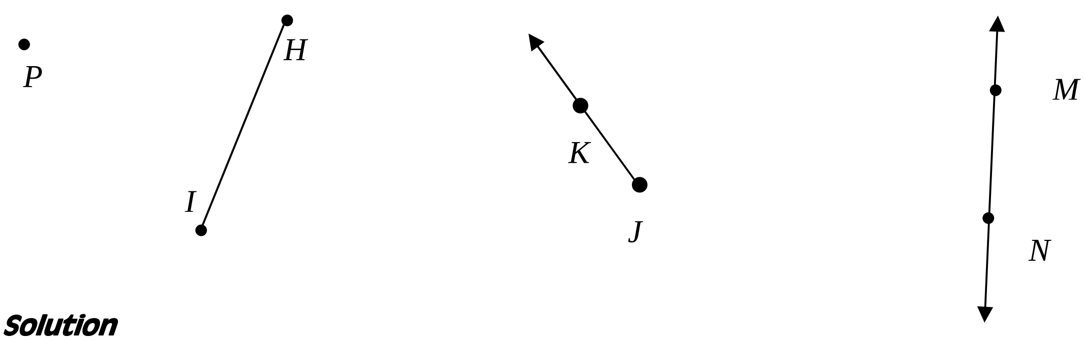
- A **line segment** has two points. Each point is an endpoint.

  - The segment above has no arrows. Points *A* and *B* are its endpoints. The notation for a line segment is

$$\overline{XY} \text{ or } \overline{YX} .$$

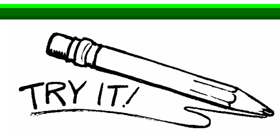
Example

Classify the following figures, then write them using the proper notation.



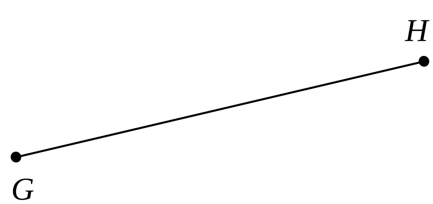
Solution

From left to right, we have point *P*, line segment $\overline{HI}$, ray $\overrightarrow{JK}$, and line $\overleftrightarrow{MN}$.



Classify each figure by writing point, line, ray, or line segment next to it. Then, name each using the proper notation.

1.



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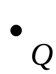
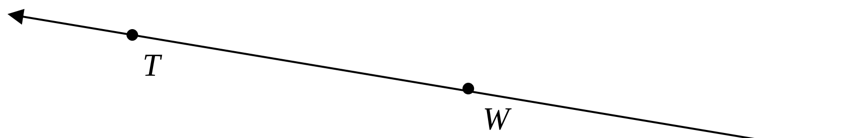
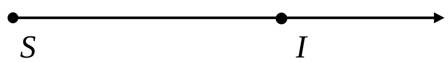


3.

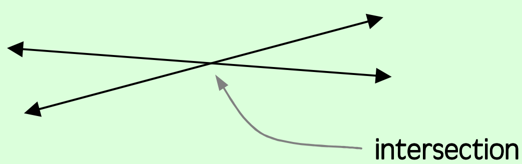


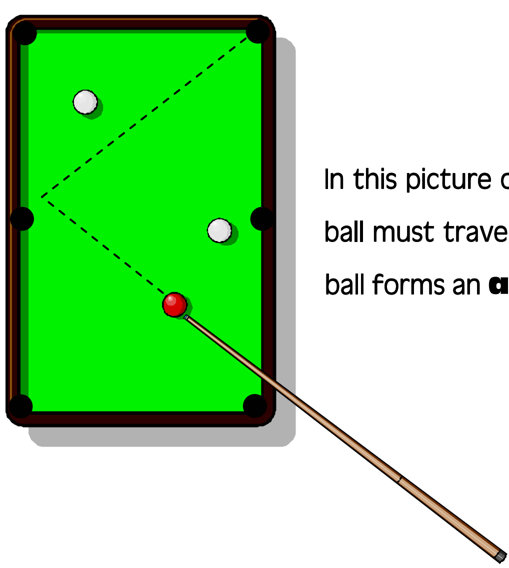
4.



When you are in a car, you have to stop at red lights at every **intersection**. With roads, an intersection is when the paths of two roads cross. The same is true with geometry.

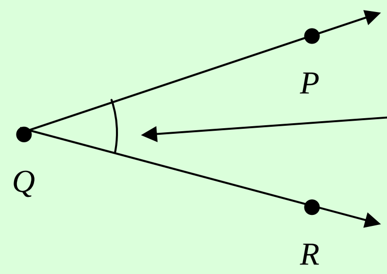
- The point where a line, a segment, or a ray crosses another line, segment, or ray, is called an **intersection**.





In this picture of a pool table, the dotted line represents the path the red ball must travel in order to drop in the top right pocket. The path of the ball forms an **angle**.

- An **angle** is formed by two *lines*, *rays*, or *segments* that meet at a common point. The common point is called the **vertex**.



We use this arc to help show the angle we are focusing on.

  - This angle is formed by two rays with vertex *Q*. There are three ways to name the angle:

$\angle PQR$

$\angle RQP$

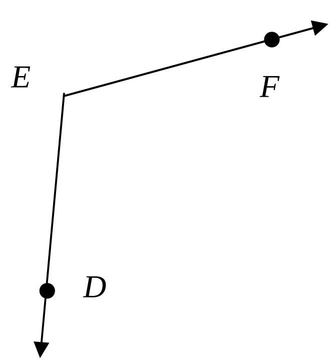
$\angle Q$

The symbols $\angle$, $\measuredangle$, and $\sphericalangle$ all mean angle.

When naming angles, make sure the vertex is the middle letter.

Example

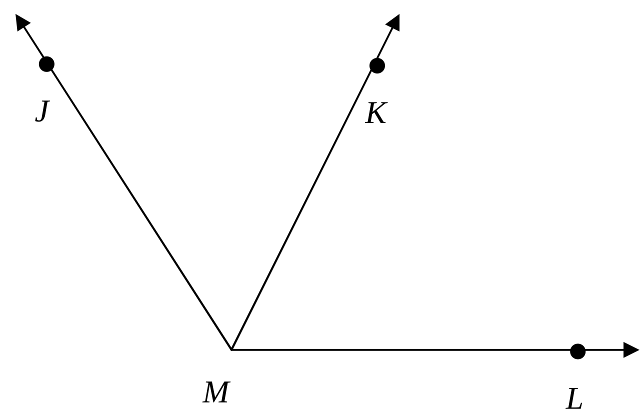
Determine the vertex, and name the angle in three ways.



Solution

Even though there is no point drawn where the two rays meet, we still call *E* the vertex of the angle. This means that *E* is the middle letter when we name the angle. We can name the angle $\angle DEF$, $\angle FED$, or $\angle E$.

Look at this example. If we wanted to name the angles below, we cannot name them $\angle M$. There is more than one angle with *M* as its vertex. It is not clear if $\angle M$ is referring to $\angle JMK$, $\angle KML$, or $\angle JML$.



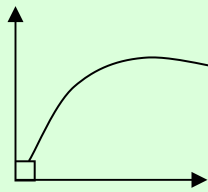
Each angle has a measure. The unit of measure for angles is called **degrees**.

- **Degrees** $(^\circ)$ are a unit of measure for angles. They are different from degrees Fahrenheit and degrees Celsius, because those measure temperature.

Take a look around you. Angles are everywhere! To make the chair you're sitting on, a furniture maker had to measure many angles. The angle measurements told him how to fit the chair together. Just as angles are in furniture and buildings, they occur everywhere in nature, too! The "golden angle", about 137.5°, is found in many plants. It is the angle between the seeds of a sunflower.

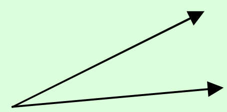
Angles come in all sizes. Scientists and mathematicians classify or organize them based on their measures.

- An angle whose measure is <u>exactly</u> 90° is a **right angle**.



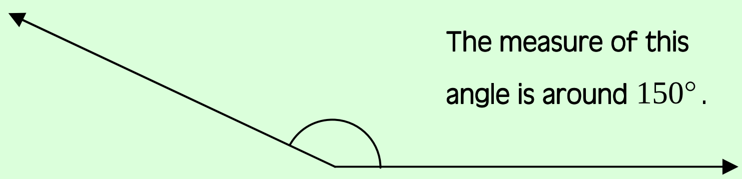
We use the little box to show the angle is a right angle.

- An angle whose measure is <u>less than</u> 90° is an **acute angle**.



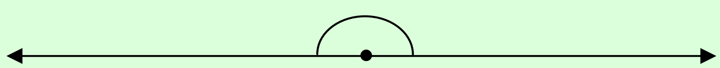
The measure of this angle is around 20°.

- An angle whose measure is <u>between</u> 90° and 180° (but not equal to 90° and 180°) is an **obtuse angle**.



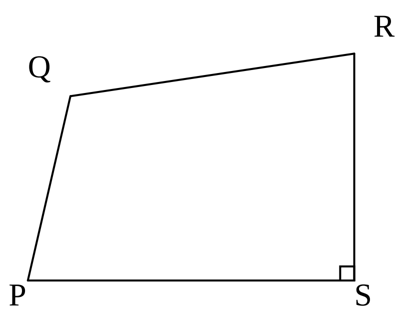
The measure of this angle is around 150°.

- An angle whose measure is <u>exactly</u> 180° is a **straight angle**.



Example

Given figure PQRS,



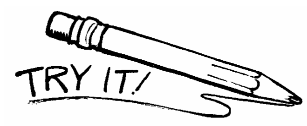
Classify the angles as acute, right, or obtuse.

A <u>benchmark</u> is used to compare things and classify them!

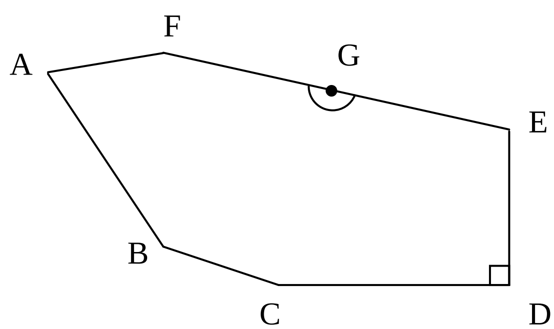
Solution

By looking at the figure, you can tell that $\angle S$ is a right angle. Not only does it have the right angle symbol, it also looks as if it is $90°$. Because of this, a right angle can be used as a <u>benchmark</u> to tell whether angles are greater than, or less than $90°$.

Compared to $\angle S$, we can see that $\angle P$ and $\angle R$ are acute, because their measures are less than $90°$. $\angle Q$ is obtuse because its measure is greater than $90°$.



Given the following figure,



Classify the following angles as acute, right, obtuse, or straight.

5. $\angle A$: ____________________

6. $\angle CBA$: ____________________

7. $\angle CDE$: ____________________

8. $\angle AFE$: ____________________

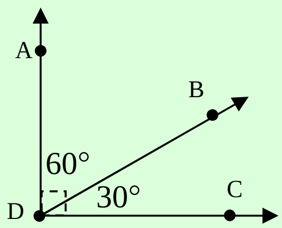
9. $\angle FGE$: ____________________

10. $\angle E$: ____________________

11. $\angle BCD$: ____________________

In geometry, right angles, and straight angles are very important.

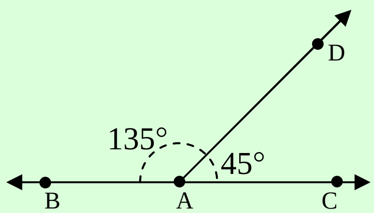
- **Complementary angles** are a pair of angles whose measures have a sum of $90°$.



$\angle ADB$ and $\angle BDC$ are complementary, since

$m\angle ADB + m\angle BDC = 60° + 30° = 90°$.

- **Supplementary angles** are a pair of angles that, together, form a straight line. Supplementary angles' measures have a sum of $180°$.



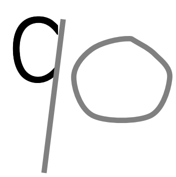
$\angle BAD \quad \angle DAC$ are supplementary, since

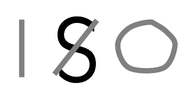
$m\angle BAD + m\angle DAC = 135° + 45° = 180°$

Here is a great way to remember complementary and supplementary. Look at the first letters of <u>c</u>omplementary and <u>s</u>upplementary.

C S

The "c" for complementary can be made into a 9 for $90°$. The "s" for supplementary can be made into an 8 for $180°$.

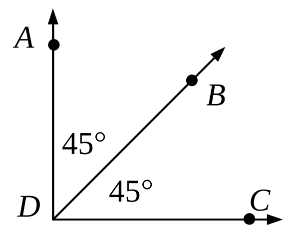


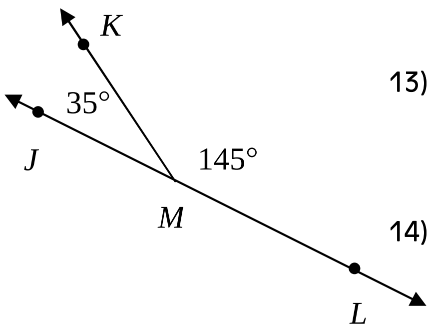


Another way to remember it is that $90°$ comes before $180°$, just as "c" comes before "s" in the alphabet.

Determine whether the angles are complementary, supplementary, or neither. Show the work you used to make your decision.

12. $\angle ADB$ and $\angle BDC$



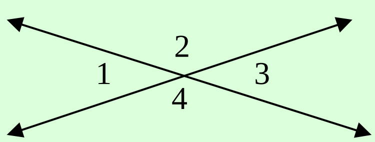


13) $\angle JMK$ and $\angle KML$

14) $\angle JML$ and $\angle KML$

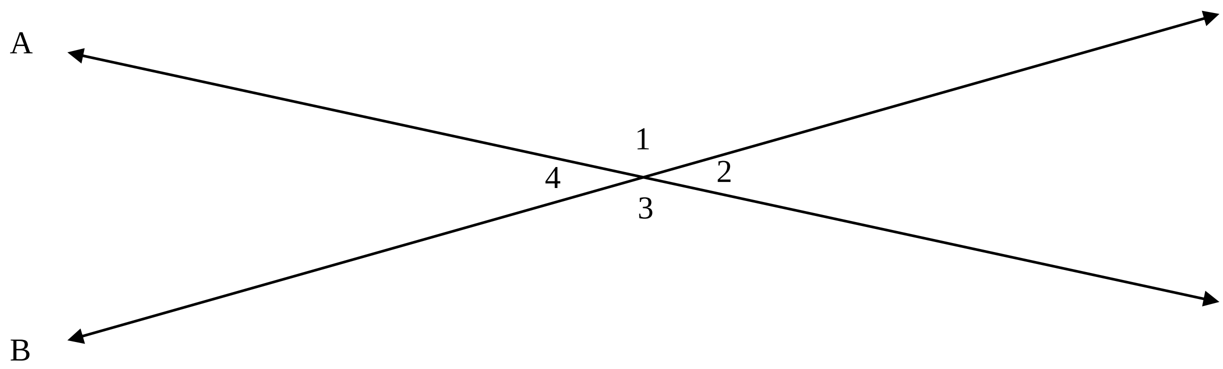
There is one more type of angle to discuss.

- When two lines intersect, the angles opposite each other are called **vertical angles**.



In the diagram, $\angle 1$ and $\angle 3$ are vertical angles. So are $\angle 2$ and $\angle 4$.

Let's take a closer look at vertical angles. Lines $\overleftrightarrow{A}$ and $\overleftrightarrow{B}$ intersect to form $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$.



We notice that $\angle 1$ and $\angle 2$ are <u>supplementary</u>. Since they are supplementary, the sum of their measures is $180°$. That means that

$$m\angle 1 + m\angle 2 = 180°$$

We can use this fact to say that the measure of angle 1 is,

$$m\angle 1 = 180° - m\angle 2$$

We also notice that $\angle 3$ and $\angle 2$ are supplementary. Because they are supplementary, the sum of their measures is $180°$ as well. Once again, this means that

$$m\angle 3 + m\angle 2 = 180°$$

We can use this to say that the measure of angle 3 is,

$$m\angle 3 = 180° - m\angle 2$$

Look at this. We just showed that the measures for angles 1 and 3 are the same.

$$m\angle 1 = 180° - m\angle 2$$

$$m\angle 3 = 180° - m\angle 2$$

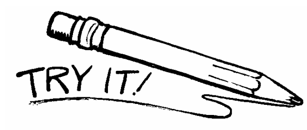
Because the measures of angles 1 and 3 are both equal to the same thing, we can say that

$$m\angle 1 = m\angle 3$$

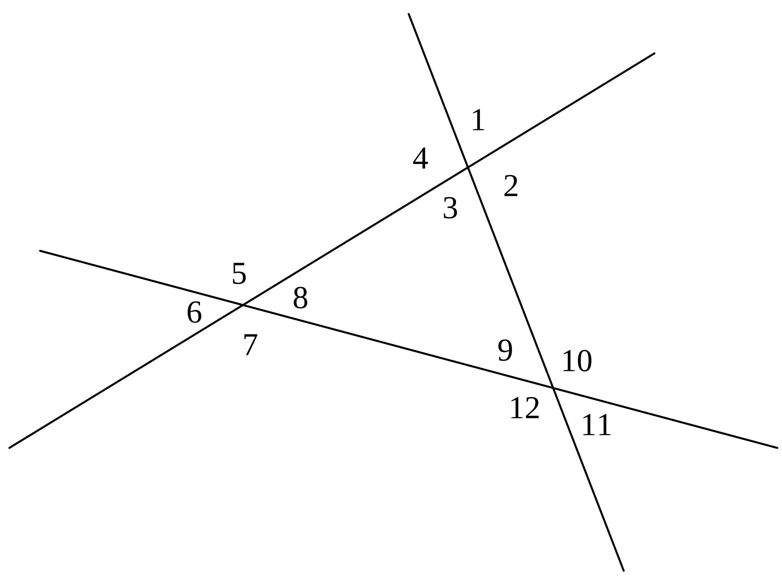
This method will always work. If you found it confusing, that's okay. Just remember this fact.

FACT

Vertical angles always have equal measure!



The following figure shows the intersections of 3 line segments to form angles 1 through 12. Identify all vertical angles, then state which angles are equal in measure.

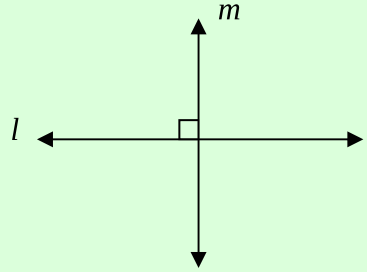


15. Vertical angles:

16) Angles equal in measure:

Sometimes, the relationship between two lines has a special name.

- Two lines are **perpendicular**, if they intersect to form four right angles.



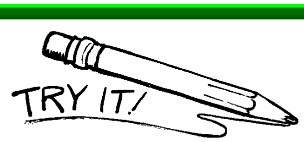
Line *l* is perpendicular to line *m*. We write this as,

$$\overleftrightarrow{l} \perp \overleftrightarrow{m}.$$

Notice that we only need to write one " □ " right angle symbol. This is due to our supplementary angle property. See if you can understand how this works yourself.

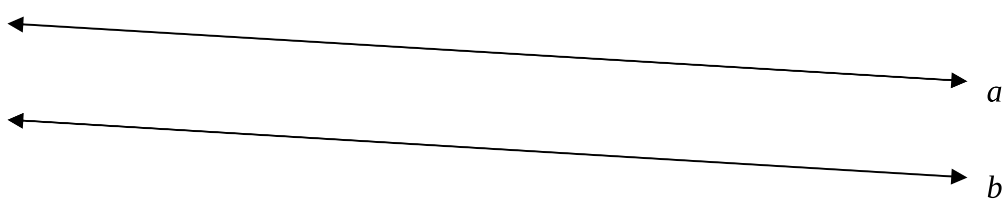
- Lines, segments, or rays are **parallel** if, when extended forever, they never touch.

Lines $\overleftrightarrow{p}$ and $\overleftrightarrow{q}$ are parallel. To show this, we write $\overleftrightarrow{p} \parallel \overleftrightarrow{q}$.

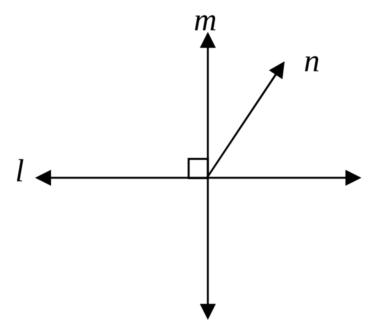


Decide whether the following lines, line segments, or rays are perpendicular, parallel, or neither.

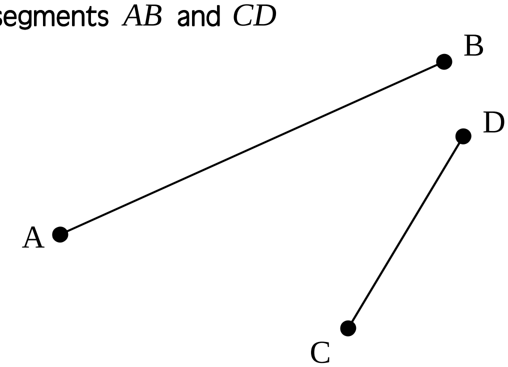
17. Lines $\overleftrightarrow{a}$ and $\overleftrightarrow{b}$



18. Lines *l* and *m*



19. Line segments $\overline{AB}$ and $\overline{CD}$



These definitions are used all the time in geometry, so be sure you are familiar with them.

Review

1. Highlight the following definitions:
   a. point
   b. line
   c. ray
   d. line segment
   e. intersection
   f. angle
   g. vertex
   h. right angle
   i. acute angle
   j. obtuse angle
   k. straight angle

l. complementary angles

m. supplementary angles

n. vertical angles

o. perpendicular

p. parallel

2. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

____

____

____

____

Practice Problems

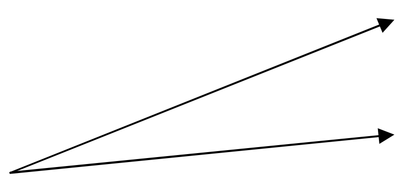
Math On the Move Lesson 17

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 17, Set A and Set B.

Set A

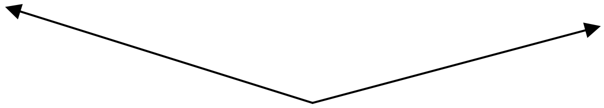
1. Are the following angles acute, obtuse, right, or straight?

a)

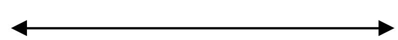


b)

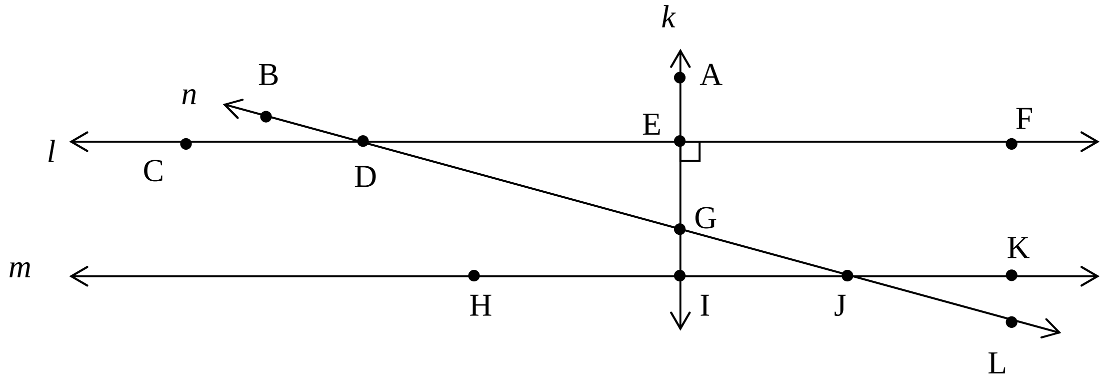
c)



d)



2. Identify the perpendicular and parallel lines. Then state which angles are vertical angles.

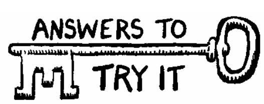


Set B

1. Look around your house, or outside. Find 2 examples of each of the following angles:
   - acute angle
   - right angle
   - obtuse angle
   - straight angles

Now look for the following lines:

- intersecting lines or line segments
- perpendicular lines
- parallel lines



1. line segment $\overline{GH}$ or $\overline{HG}$
2. point Q
3. line $\overleftrightarrow{TW}$ or $\overleftrightarrow{WT}$
4. ray $\overrightarrow{SI}$
5. acute
6. obtuse
7. right
8. obtuse

9. straight

10. obtuse

11. obtuse

12. complementary $m\angle ADB = 45°$; $m\angle BDC = 45°$ $45° + 45° = 90°$

13. supplementary $m\angle JMK = 135°$; $m\angle KML = 45°$ $135° + 45° = 180°$

14. neither

15. $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$, $\angle 5$ and $\angle 7$, $\angle 6$ and $\angle 8$, $\angle 9$ and $\angle 11$, $\angle 10$ and $\angle 12$

16. Angles with the same measure are all the vertical angle pairs.

17. Parallel

18. Perpendicular

19. Neither

End of Lesson 17