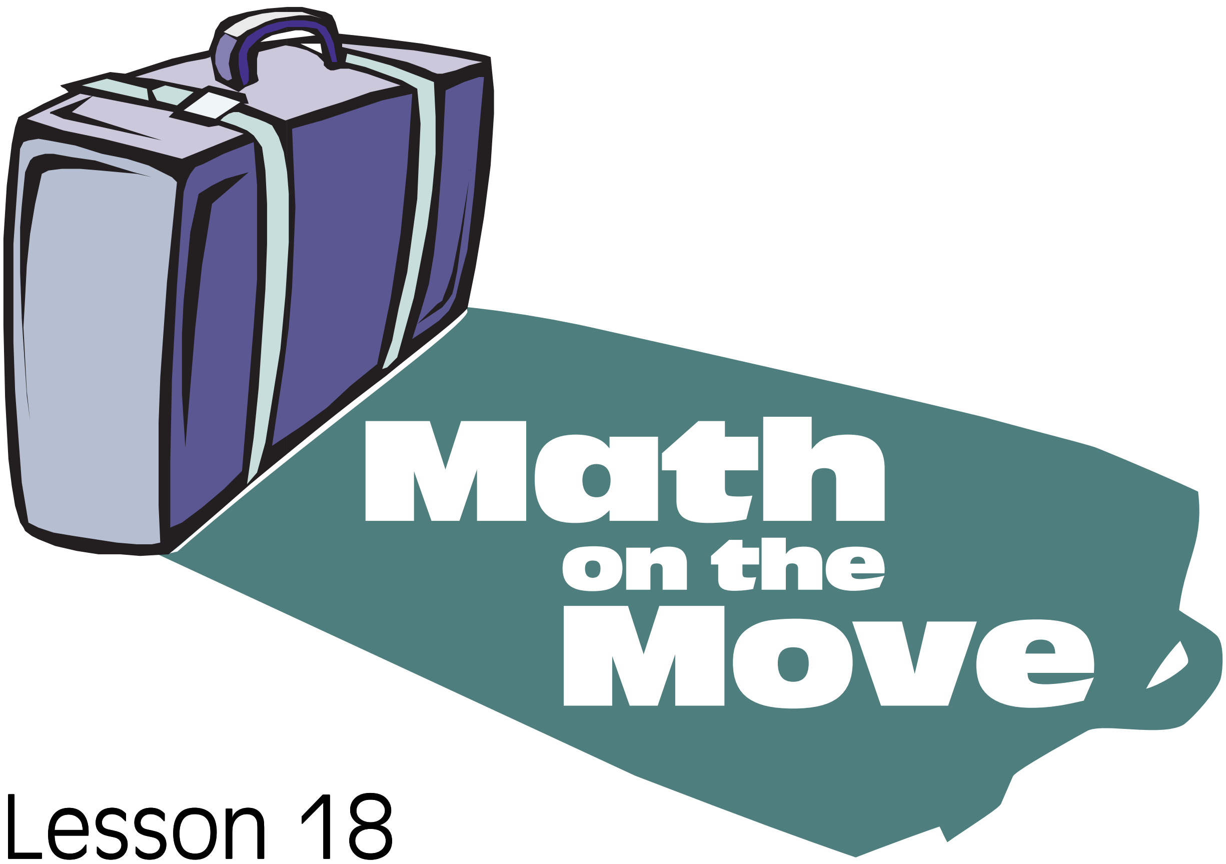
Student Name: ______________________________

Date: ______________________________

Contact Person Name: ______________________________

Phone Number: ______________________________



Lesson 18

Quadrilaterals

Objectives

- Understand the definition of a quadrilateral
- Distinguish between different types of quadrilaterals
- Find the perimeter and area of a quadrilateral
- Determine when polygons are similar
- Use properties of similar polygons to solve problems

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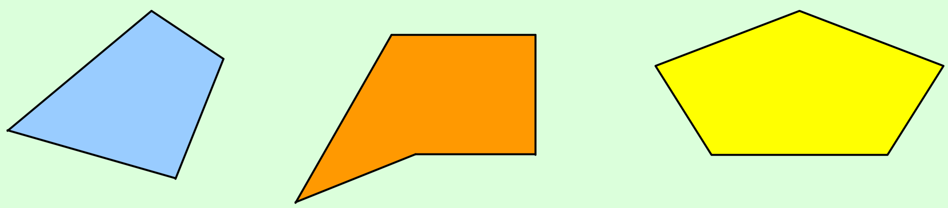
Your friend Juana invites you over to her house to go swimming in her pool. She talks about how her pool is a **polygon** shaped like a **quadrilateral**. You say to her, "What is a **polygon**?"

- A **polygon** is a closed figure made by joining line segments. Each line segment joins two others, forming a vertex.
    - The following are examples of polygons.



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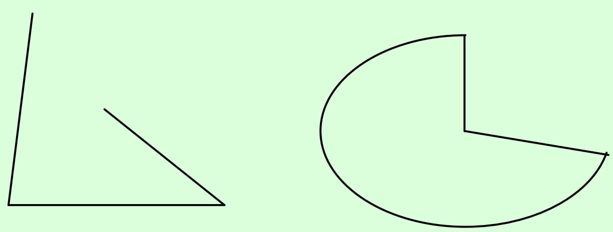
    - The following are examples of non-polygons.

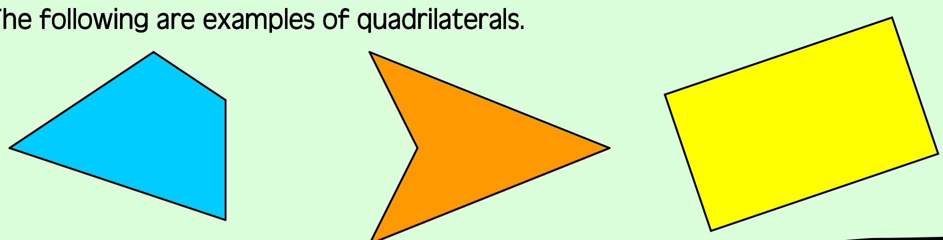


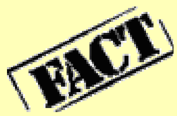
Not closed

Not line segments

After Juana explains what a polygon is, you ask her, "Okay, so what's a **quadrilateral**?"

- A **quadrilateral** is a <u>four-sided</u> polygon. It is a polygon made of four line segments that form four <u>vertices</u> or angles. The sum of all the angles in a quadrilateral is $360°$.
    - The following are examples of quadrilaterals.





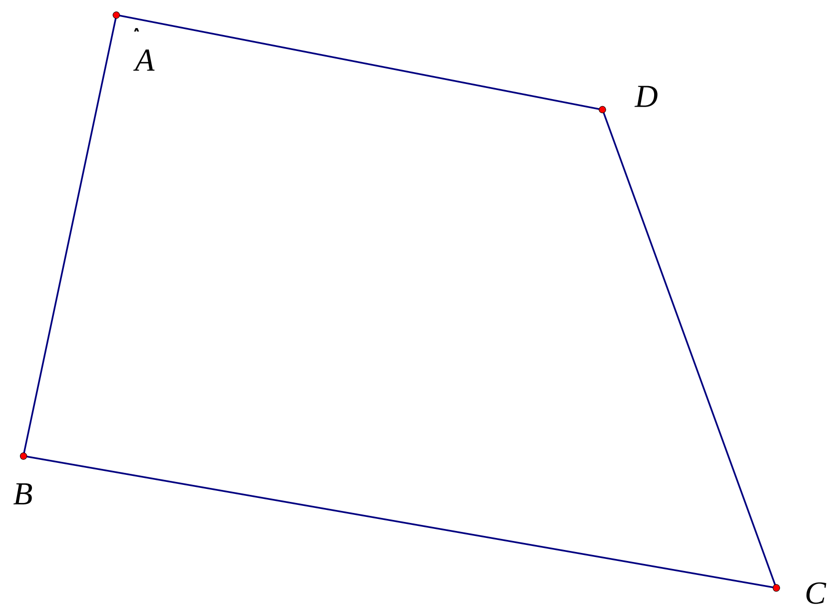
"Vertices" is the plural form of the word "vertex".

Think Back

We discussed a vertex when we talked about angles. A vertex is formed when two lines, rays, or segments intersect each other.

Example

Find the measure of each of the vertex angles in the following quadrilateral using a protractor. Then find the sum of all the angles.



Solution

Using a protractor, we can measure each vertex.

$$m\angle A = 91^\circ$$

$$m\angle B = 88^\circ$$

$$m\angle C = 60^\circ$$

$$m\angle D = 121^\circ$$

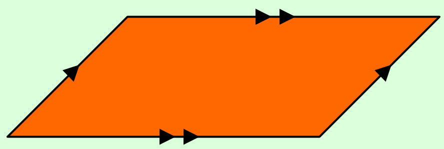
Now we can add up all the angles.

$$m\angle A + m\angle B + m\angle C + m\angle D = 91^\circ + 88^\circ + 60^\circ + 121^\circ = 360^\circ$$

As expected, the sum of all the vertex angles is 360°.

After explaining what a quadrilateral is, Juana goes into more detail about her pool. She states, "My pool is shaped like a **parallelogram**." You say to her, "Wait a minute. You just told me your pool was a quadrilateral. How can it also be a parallelogram?"

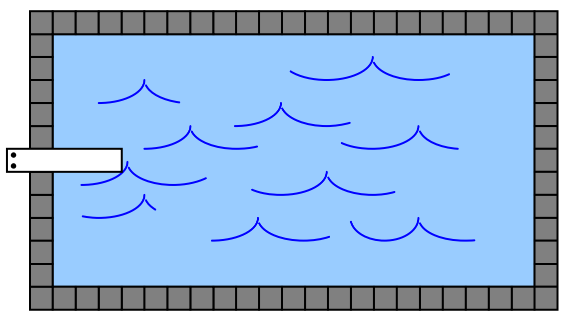
- A **parallelogram** is a quadrilateral whose opposite sides are parallel. Notice that the word parallel is written in the word parallelogram.
    - o The following is an example of a parallelogram.



We use the arrows to show that the lines are parallel. The segments with single arrows are parallel to each other, while the segments with double arrows are parallel to each other.

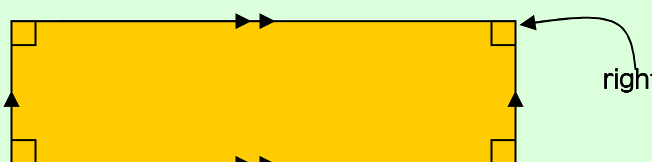
So, a parallelogram is a quadrilateral because it has four sides.

The information about Juana's pool is interesting. You decide that you have to go. When you arrive at Juana's house, she shows you her pool. It looks like this.



You look at Juana and say, "This isn't a parallelogram. This is a **rectangle**."

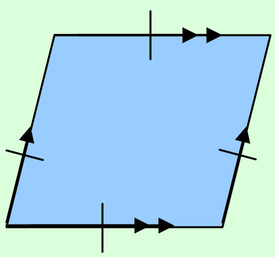
- A **rectangle** is a parallelogram where each vertex forms a right angle. Sides touching each other are perpendicular. As with a parallelogram, both sets of opposite sides of a rectangle are parallel.
    - o The following is an example of a rectangle.



Juana points out that her pool is both a rectangle and a parallelogram. She states, "A rectangle has both sets of opposite sides parallel to each other." That is how we defined a parallelogram. So, a rectangle is always a parallelogram. However, a parallelogram *is not always* a rectangle."

After swimming, you become curious about quadrilaterals. You wonder if there are any other interesting properties about them. The properties discussed so far deal with perpendicular and parallel sides. The last property we have to discuss is the length of the sides. In a parallelogram, both sets of opposite sides are equal. In a **rhombus**, all sides are equal.

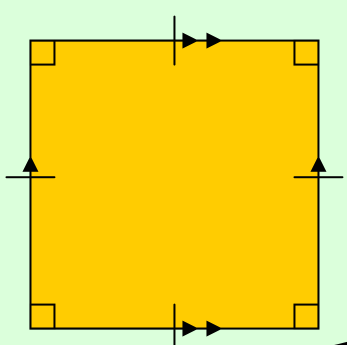
- A **rhombus** is a special parallelogram that has four equal sides. All four sides have the same measure. As with a parallelogram, both sets of opposite sides are parallel in a rhombus.
    - The following is an example of a rhombus.

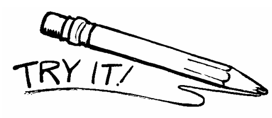


We use the single hash marks on each side to show that each side is the same length.

Juana continues, "The most familiar form of the rhombus is the **square**."

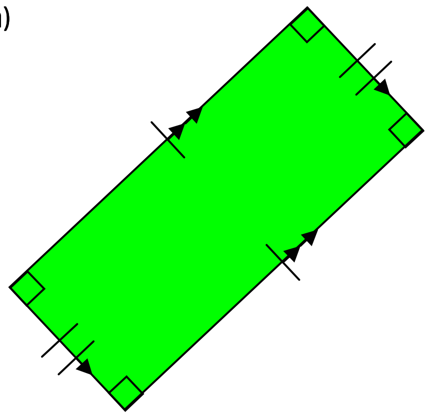
- A **square** is a rhombus where each vertex forms a right angle. All sides are equal, and the sides that are next to each other are perpendicular. As with a parallelogram, both sets of opposite sides are parallel in a square.
    - The following is an example of a square.



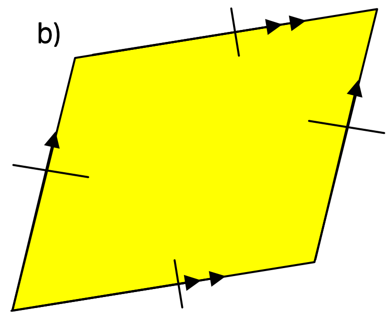


1. Classify the following quadrilaterals in as many ways as possible.

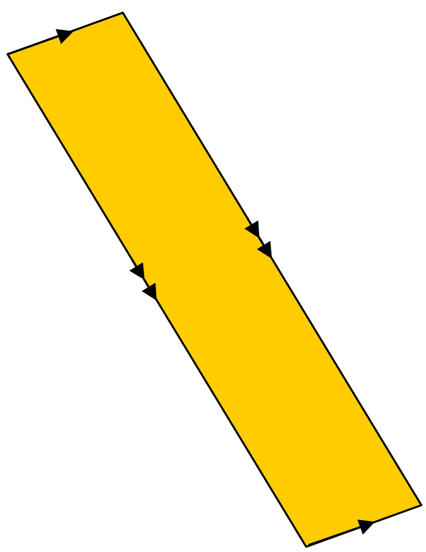
a)



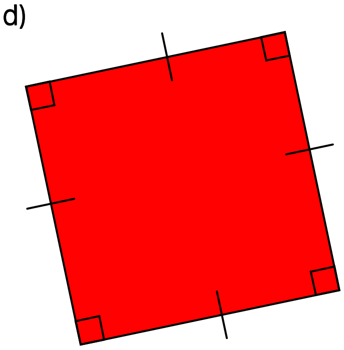
b)



c)

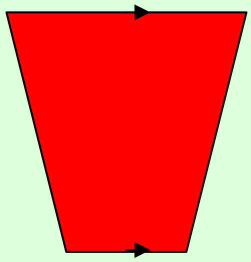


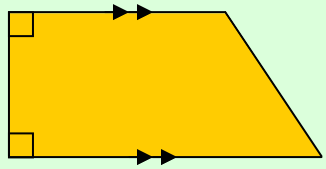
d)



"Wow, this is all very interesting!" you exclaim. "So, this means that a square is a rhombus and a parallelogram?" She answers, "Yes, but don't forget a square is also a rectangle!" You state, "It seems as if all these special quadrilaterals are parallelograms." Juana replies, "It certainly does, but it's not true. There is also the **trapezoid**."

- A **trapezoid** is a quadrilateral that has <u>only</u> one set of opposite sides parallel. A trapezoid is not a parallelogram, nor is a parallelogram a trapezoid.
    - The following are examples of trapezoids.

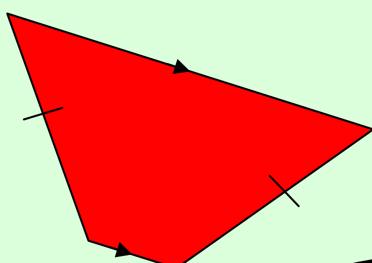




Even though the second example has two right angles, it is still not a rectangle. All four angles of a rectangle are right angles. Unlike parallelograms, the opposite sides of a trapezoid do not necessarily have the same measure.

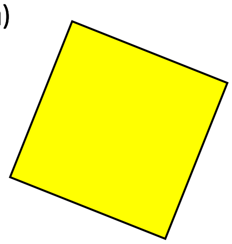
Juana continues by telling you about a special type of trapezoid called the **isosceles trapezoid**.

- An **isosceles trapezoid** is a trapezoid that has two sides with the same measure. The sides that are not parallel to each other have the same measure.
    - The following is an example of an isosceles trapezoid.



2. Determine whether the following shapes are trapezoids, isosceles trapezoids, or neither.

a)



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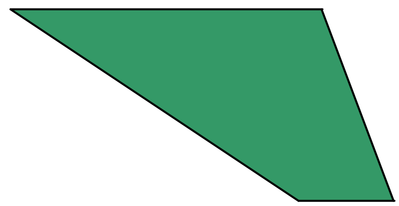
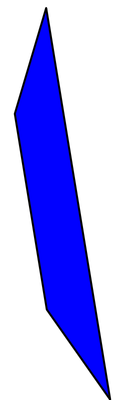
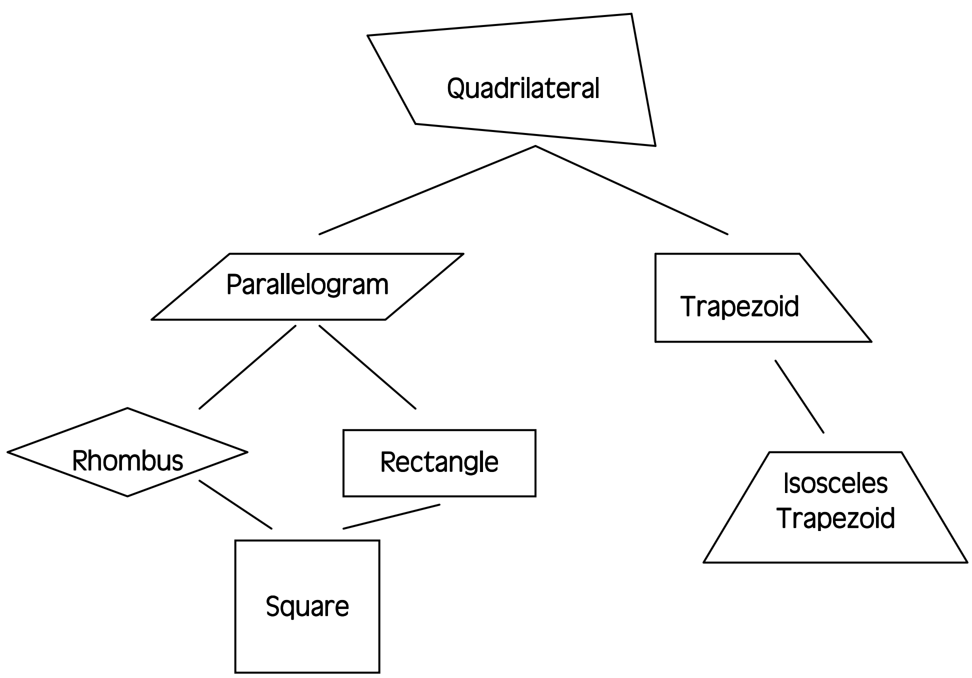


c)



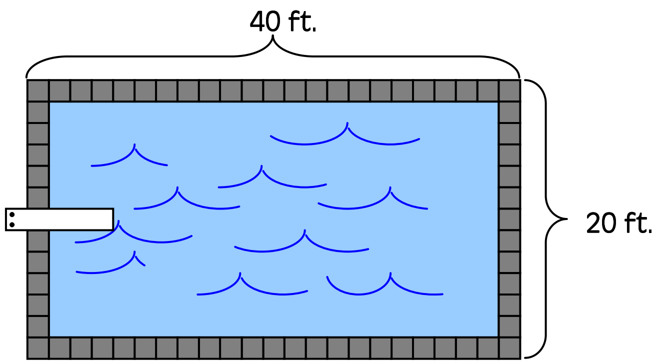
Let's use the following chart to help us classify quadrilaterals.



Juana tells you more about her pool. She talks about how she wants to put new bricks around her pool. In order to do this, she needs to determine the **perimeter** around her pool.

- **Perimeter** is the distance around a polygon. It is the sum of the lengths of the sides.

Juana tells you her pool is 20 ft. by 40 ft.



FACT

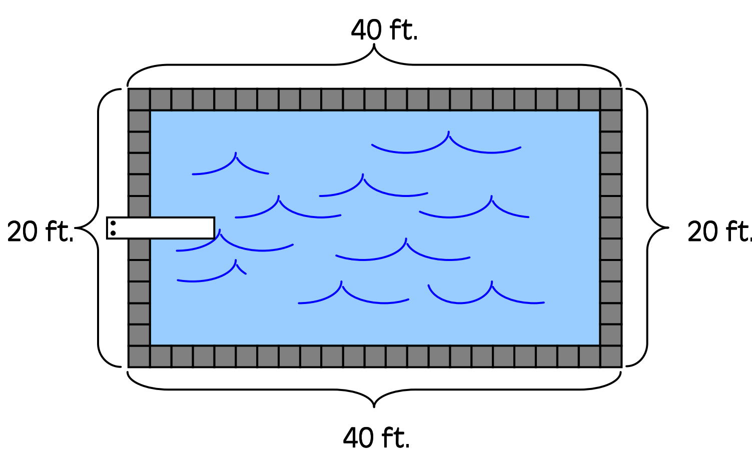
When we say the dimensions of a rectangle aloud, we say the length of one side by the length of the other side.

Dimensions of a rectangle are often called the length and the width, or the base and the height.

So, we know the length of the pool is 40 ft. and the width is 20 ft. We need to find the sum of the lengths of all the sides of the rectangle. Since a rectangle is a parallelogram, the set of sides opposite each other are the same length. This means that we know the length of all four sides in the rectangle.

FACT

In a parallelogram, the set of sides opposite each other have the same length.



Since we know the length of all four sides, we can add them up.

$$40 + 40 + 20 + 20 = 120$$

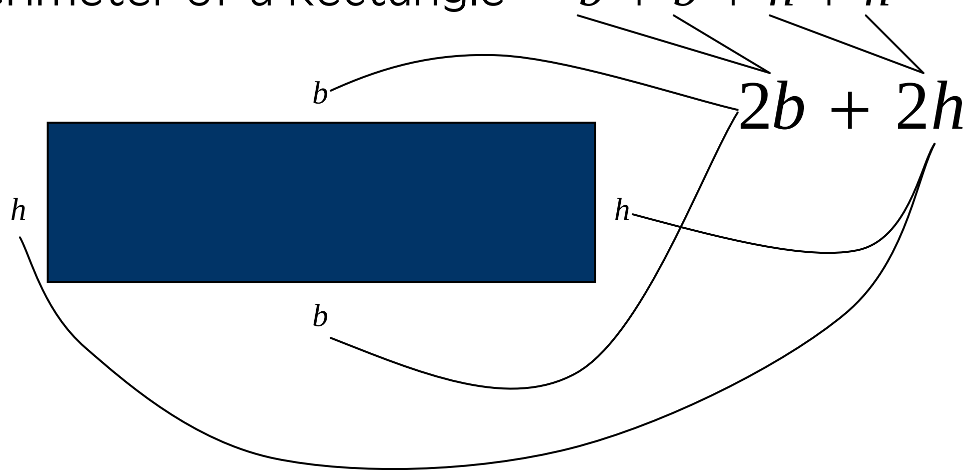
Lastly, we need to include units. The perimeter of the pool is 120 ft.

After finding the perimeter of the pool, you realize that there is a shortcut for finding the perimeter of a rectangle. Since both pairs of opposite sides are the same length, we simply add them twice. Remember that repeated addition is the same as multiplication. So,

$$40 + 40 + 20 + 20 = 2(40) + 2(20) = 120$$

This is true for all rectangles, so

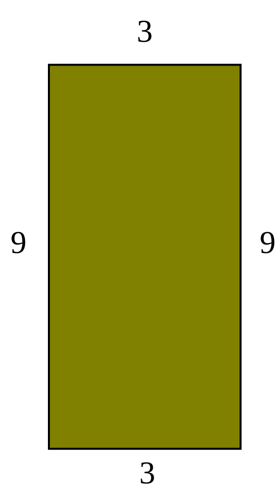
$$\text{Perimeter of a Rectangle} = b + b + h + h = 2b + 2h$$



Notice that we are using variables to represent the base and height of the rectangle. We do this to make a general formula for finding the perimeter of any rectangle.

Example

Find the perimeter for the following rectangle.



Solution

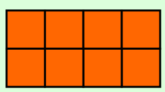
Let's try out our new formula for perimeter, we said that

$P = 2b + 2h$. We will say $b = 3$ and $h = 9$.

$$\begin{aligned} P &= 2(3) + 2(9) \\ &= 6 + 18 \\ &= 24 \text{ units} \end{aligned}$$

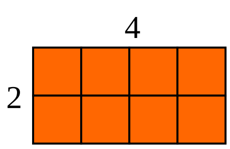
The perimeter of Juana's pool is 120 ft. Now, she wants to find the **area** of her pool.

- **Area** is the number of square units (units^2) an object takes up, such as square feet (ft.^2) or square meters (m^2).

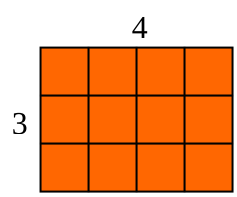


This object has an area of 8 units^2

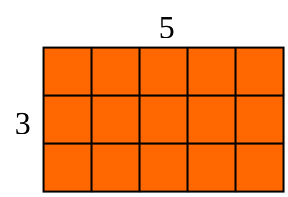
To find the area of an object, we can simply add up the total number of square units the object takes up. This seems as if it would take a long time. Since we found a formula for perimeter, let's see if we can find one for area.



Area = 8

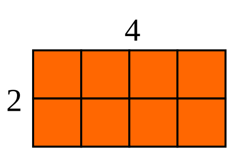


Area = 12

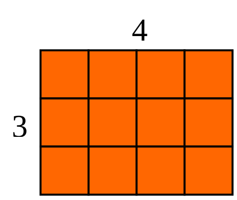


Area = 15

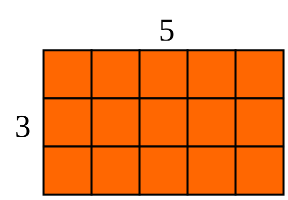
Can you see the pattern developing with these rectangles?



Area = $2 \times 4 = 8$



Area = $3 \times 4 = 12$



Area = $3 \times 5 = 15$

$$\text{Area of a Rectangle} = \text{base} \times \text{height} = b \times h$$

Based on this formula, the area of Juana's pool is

$$A = b \times h$$

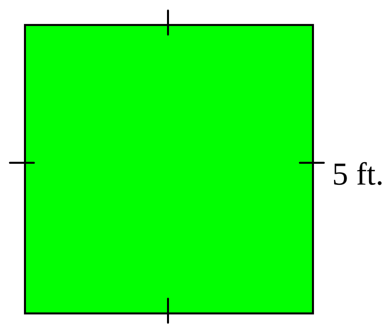
$$A = 40 \times 20$$

$$A = 800 \text{ sq. ft. or } 800 \text{ ft.}^2$$

All squares are rectangles. Therefore, the formulas for perimeter and area of a rectangle can be used for squares as well!

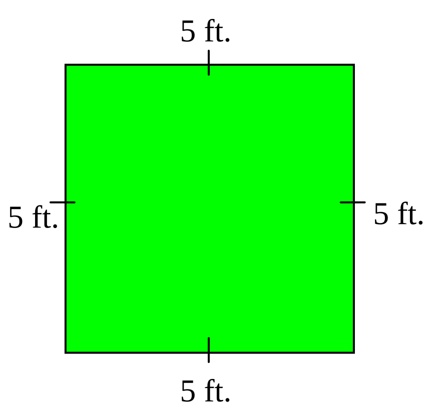
Example

Find the perimeter and area of the following square.



Solution

We are given a square, but we are only given the length of one side. The formulas we have for area and perimeter require two dimensions. We must use our knowledge of squares to solve this problem. A square is a rectangle whose sides are equal in length. So, we can fill in the length of the missing sides on the square.



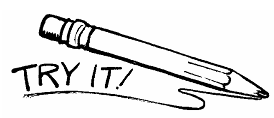
Now we can use the formulas for perimeter and area.

$$\begin{aligned} P &= 2b + 2h \\ &= 2(5) + 2(5) \\ &= 10 + 10 \\ \text{Perimeter} &= 20 \text{ ft.} \end{aligned}$$

$$\begin{aligned} A &= bh \\ &= (5)(5) \\ \text{Area} &= 25 \text{ ft.}^2 \end{aligned}$$

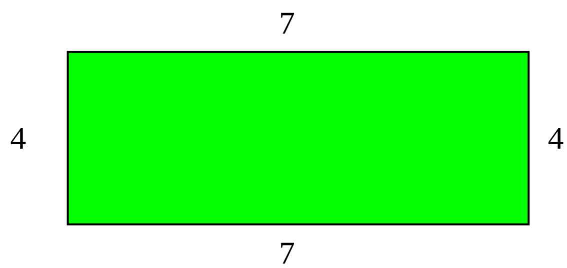
Think Back

When letters and numbers are written next to each other with no symbol between them, it means multiplication.



3. Determine the <u>perimeter and area</u> for the following rectangles.

a)



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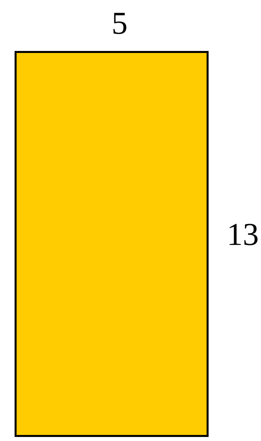
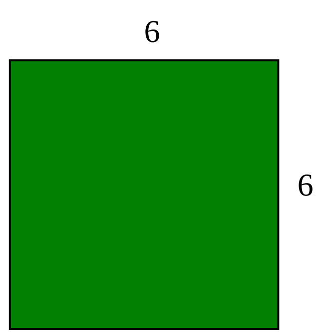
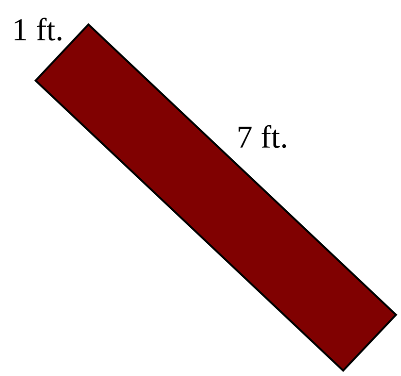


c)



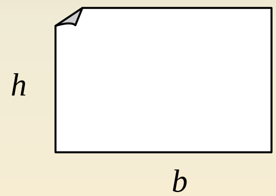
d)



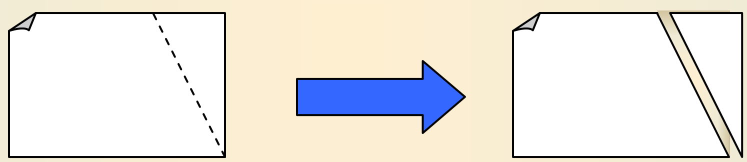
Rectangles and squares could easily be divided into square units, but what about parallelograms?

Find out for yourself!

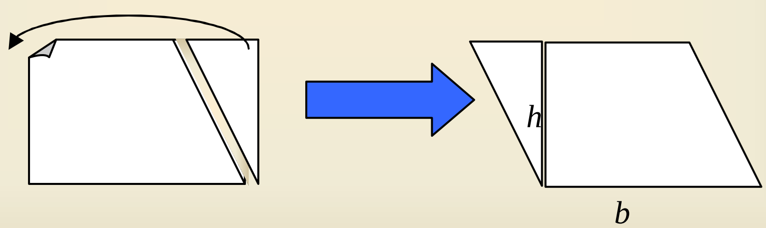
Step 1: Take a rectangular sheet of paper and find its area.



Step 2: Using a straightedge, draw a line from the bottom right hand corner of the paper to anywhere on the top of the paper. Then, using scissors cut along that line.



Step 3: Slide the triangle from one side of the paper to the other to form a parallelogram.



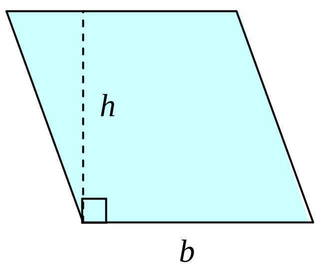
What is the area of the parallelogram? What dimensions does the parallelogram have in common with the rectangle?

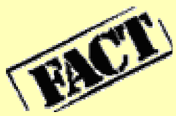
The area of the parallelogram should be the same as the rectangle, because both shapes have the same amount of paper. The dimensions that the two shapes share are the base, *b*, and the height, *h*.

We know that the area of a rectangle is $A = b \times h$. Therefore, the formula for the area of a parallelogram is equal to the formula for the area of a rectangle.

$$\text{Area of a Parallelogram} = b \times h$$

In a parallelogram, height is the length of the line segment drawn perpendicularly from base to base.

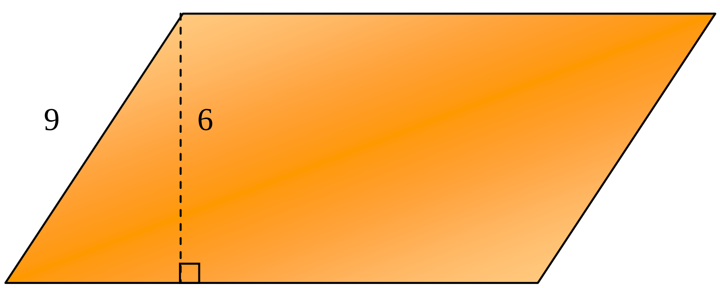




Every rhombus is a parallelogram. Therefore, the formulas for perimeter and area of a parallelogram can be used for a rhombus as well!

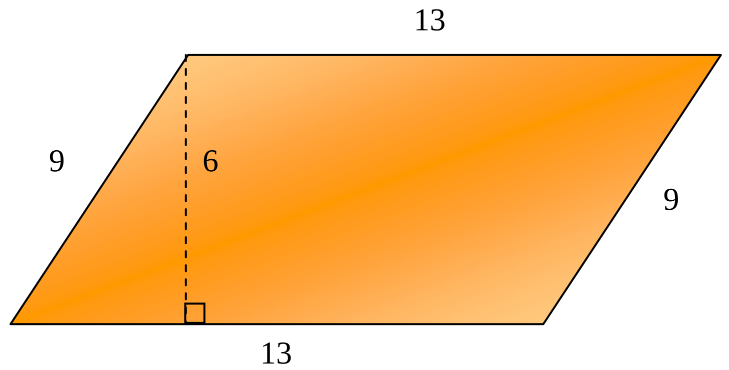
Example

Find the perimeter and area of the following parallelogram.



Solution

We are not given the length of each side of the parallelogram. We must remember that opposite sides of a parallelogram are the same lengths. Because of this, we know the lengths of the missing sides.

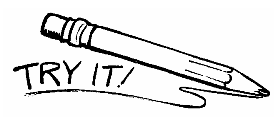


We find the perimeter by adding the lengths of the sides.

$$P = 9 + 9 + 13 + 13 = 44$$

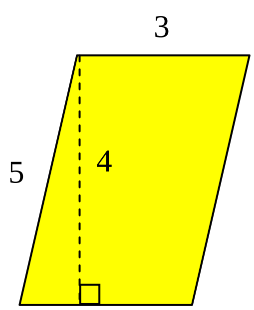
And we find the area by substituting the proper values into the formula,

$$A = b \times h = 13 \times 6 = 78 \text{ units}^2$$

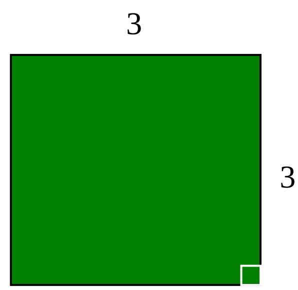


4. Find the perimeter and area of the following parallelograms.

a)



b)



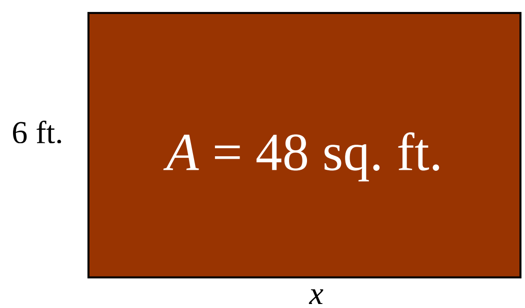
Now that we know all about quadrilaterals, we can solve missing dimension problems.

Example

Penelope wants to build a rectangular pig pen with an area of 48 sq. ft. If she wants the width of the pen to be 6 ft., what should the length of the pen be?

Solution

The best way to solve this problem is by drawing a picture. We know that the shape of the pen will be a rectangle, so we will draw a rectangle.



Since we do not know the length of the pen, we will use a variable, *x*, to represent it. We know that the formula for the area of a rectangle is $A = b \times h$, so we will plug in our values.

$$A = b \times h$$

$$48 = 6x$$

Now we can solve for the variable.

$$\frac{48}{6} = \frac{6x}{6}$$

$$8 = x$$

So, the length of the pen will be 8 ft.

Check:

$$A = b \times h$$
$$48 = 8 \times 6$$
$$48 = 48 \quad \checkmark$$

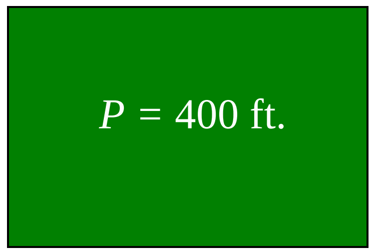
Let's try a harder one

Example

Santiago has 400 feet of fence to make a rectangular field for his horses to graze in. He wants the length of the field to be 50 ft. more than the width. What are the dimensions of the rectangular field?

Solution

First, we will draw a picture of a rectangle.



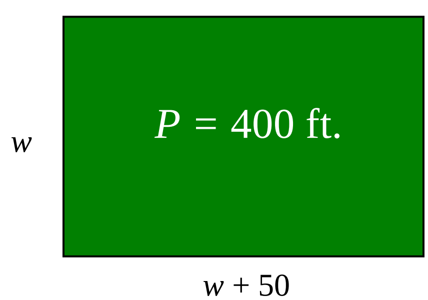
We know the perimeter of the rectangle is 400 ft., because Santiago has 400 ft. of fence. We do not know either of the dimensions of the rectangle, so we need to create let statements. The second sentence in the problem says, "He wants the length of the rectangular field to be 50 ft. more than the width."

> ***Think Back***
>
> *Our first let statement always refers to the object we know the least about. We know the least about the width of the rectangle, because we are only told that the length is 50 more than the width. Don't forget to underline key words that imply different operations.*

Let **w = width of the rectangle**

$w + 50$ = length of the rectangle

Now that we have a representation for the dimensions of the rectangle, we can draw them in the picture.



Plug those values into the perimeter formula for rectangles, and solve for the variable.

$$P = 2b + 2h$$

$$400 = 2(w + 50) + 2(w)$$

$$400 = 2w + 100 + 2w$$

$$400 = 4w + 100$$

$$-100 \qquad\quad -100$$

$$\frac{300}{4} = \frac{4w}{4}$$

$$75 = w$$

We have found *w*, but we were looking for both dimensions. We're not done!

Let ***w*** **= width of the rectangle = 75 ft.**

w **+ 50 = length of the rectangle = 75 + 50 = 125 ft.**

Lastly, we must check the answer.

<u>*Check*</u>:

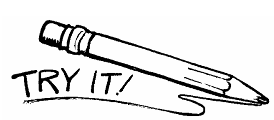
$$P = 2b + 2h$$

$$400 = 2(125) + 2(75)$$

$$400 = 250 + 150$$

$$400 = 400 \quad \checkmark$$

Try some of these word problems on your own.



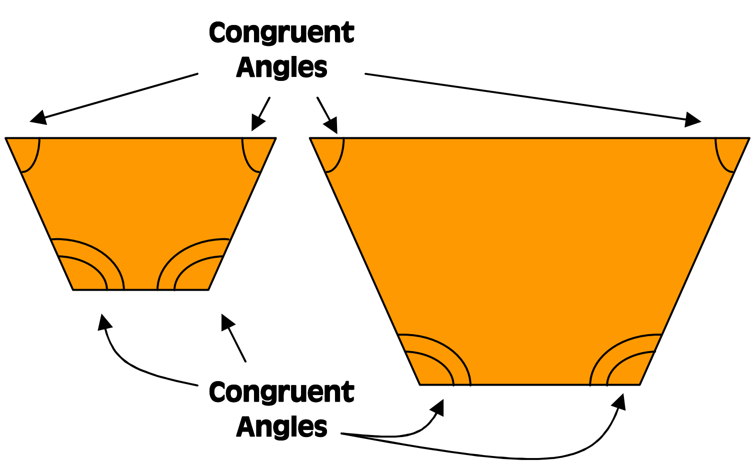
5. Mariana's bedroom is 121 sq. ft. She knows her room is shaped like a square. What are the dimensions of Mariana's bedroom?

6. Carlos walked once around a rectangular block. He found that the total distance around the block was 1800 m. If the length of the block was twice as long as the width of the block, what are the dimensions of the block?

The last thing we need to talk about is **similar polygons**.

- **Similar polygons** have the same shape, but not the same size. Corresponding angles in similar polygons are **congruent**, and corresponding sides are in proportion.
- Objects that are **congruent** have the same measure. Angles and sides of a polygon can be congruent. In a rectangle, opposite sides are congruent. Also, because all of the angles in a rectangle are right angles, all of the angles are congruent.

The two trapezoids shown below are similar.



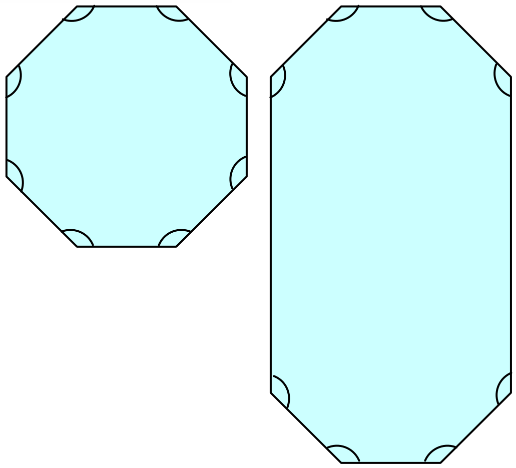
FACT

We use arcs to show when angles have the same measure. The angles with a single arc are all congruent, and the angles with a double arc are all congruent.

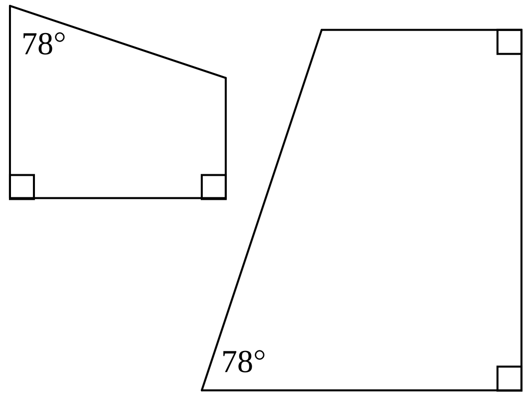
Notice the trapezoid on the left is much smaller, but the shape of the two trapezoids is consistent. The shape of a polygon is determined by the measure of its angles.

7. Determine whether the following polygons are similar or not.

a)



b)

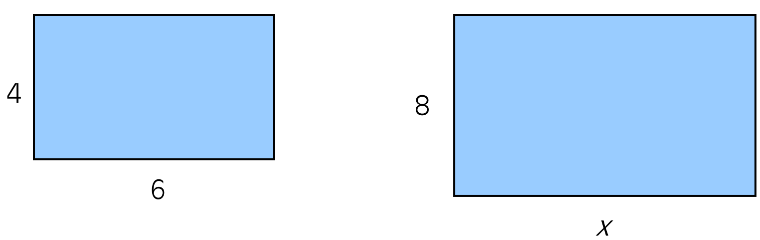


In the first example of the "try it" problems, the polygons have all the same angles, but are not the same shape. This shows that similar polygons have congruent angles, ***but*** polygons with congruent angles are not always similar.

*If corresponding angles of two **triangles** are congruent, those two **triangles** will always be similar.*

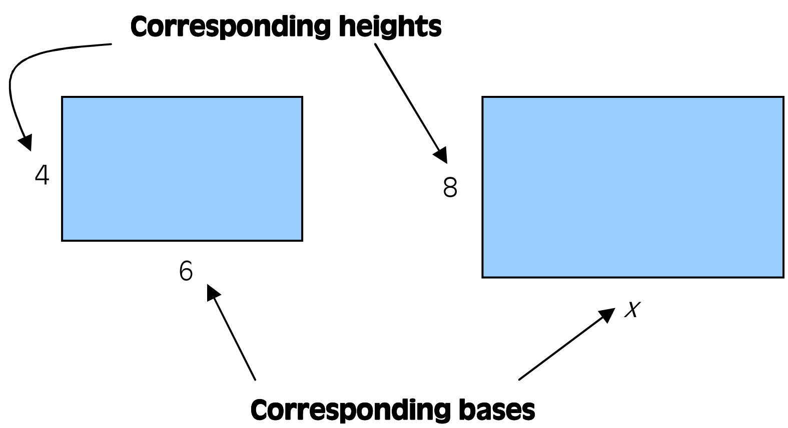
Example

The two rectangles shown are similar. Find the length of the missing side, *x*.



Solution

In our definition of similar polygons, corresponding sides are proportional. That means: the ratios between the corresponding sides are equal. When one side of a polygon matches up with the side of a similar polygon, the two sides are said to be corresponding sides. In the two rectangles, the height from the first rectangle matches up with the height from the second rectangle. Also, the base from the first rectangle matches up with the base from the second rectangle.



Now that we have determined which sides are corresponding, we can set up our proportion.

$$\frac{\text{corresponding heights}}{\text{corresponding bases}} = \frac{4}{6} = \frac{8}{x}$$

Solve it as we did all our other proportion problems:

$$\frac{4}{6} = \frac{8}{x}$$

$$\frac{4x}{4} = \frac{48}{4}$$

$$x = 12$$

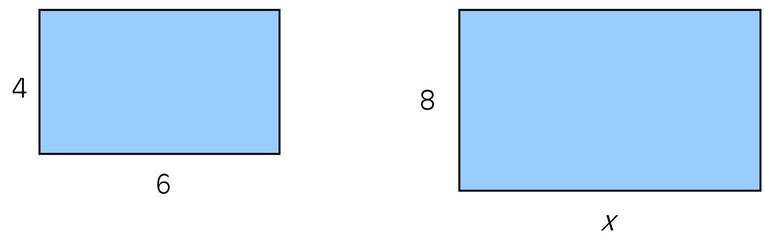
So, the length of the missing side is 9.

Always remember to check your answer.

Check: $\frac{4}{6} = \frac{8}{12}$ $\qquad \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$ $\qquad \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

Each of these fractions reduce to $\frac{2}{3}$. ✓

The great thing about proportions is that they can be set up in many different ways. Let's solve the previous example by setting up the proportion in a different way.



$$\frac{\text{corresponding bases}}{\text{corresponding heights}} = \frac{6}{4} = \frac{x}{8}$$

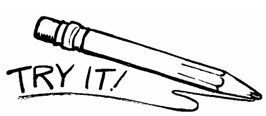
When we cross multiply, we get the same answer.

$$\frac{6}{4} \times \frac{x}{8}$$

$$\frac{48}{4} = \frac{4x}{4}$$

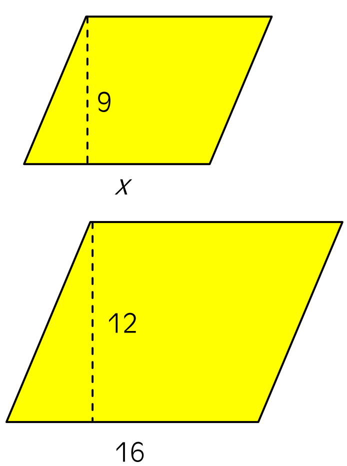
$$12 = x$$

Try some on your own!

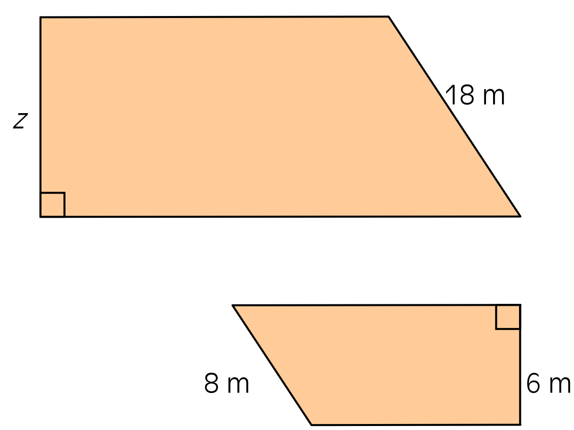


8. Find the missing values in the following similar figures.

a) Similar Parallelograms



b) Similar Trapezoids



Review

1. Highlight the following definitions:
    a. polygon
    b. quadrilateral
    c. parallelogram
    d. rectangle
    e. rhombus
    f. square
    g. trapezoid
    h. isosceles trapezoid
    i. perimeter
    j. area
    k. similar polygons
    l. congruent

2. Highlight the quadrilateral flow chart.

3. Highlight all the area and perimeter formulas.

4. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems

Math On the Move Lesson 18

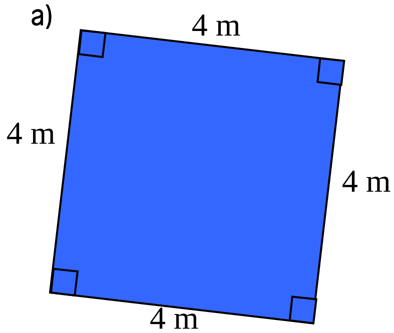
Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 18, Set A and Set B.

Set A

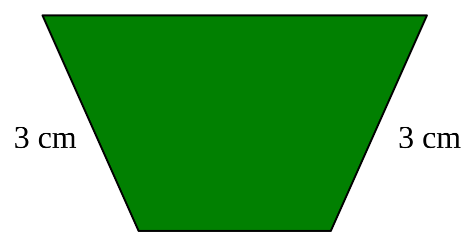
1. State whether the following statements are true or false.
   a) All squares are rectangles.
   b) All rectangles are squares.
   c) All trapezoids are quadrilaterals.
   d) All quadrilaterals are polygons.
   e) All squares are rhombuses.

2. Classify the following shapes in as many ways as possible.

a)

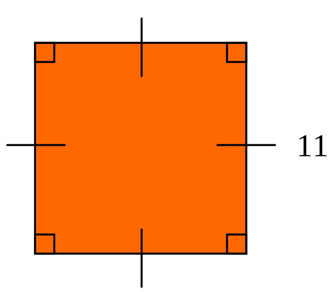


b)

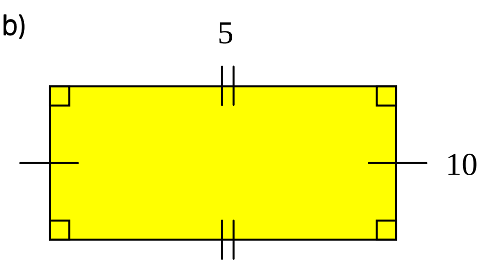


3. Find the area and perimeter of the following quadrilaterals.

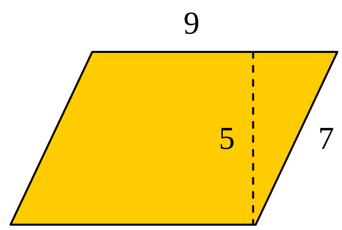
a)



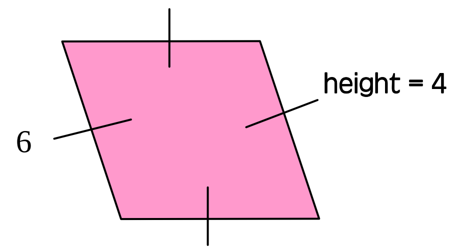
b)



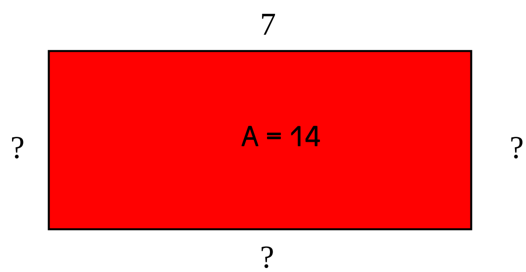
c)



d)



4. Given this rectangle, with area 14 and one side of length 7, find the lengths of the missing sides.

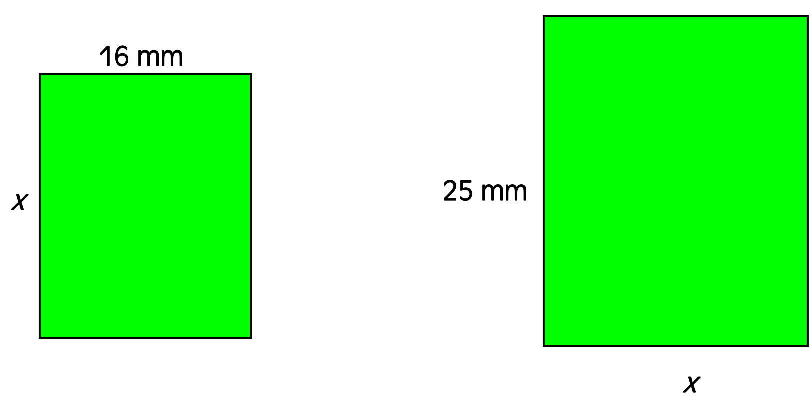


Set B

1. Find the dimensions of a square whose perimeter is equal to its area.

2. Jaime made a parallelogram with an area of 50 sq. units. If the base is twice the height, what are the dimensions of the parallelogram

3. The two rectangles below are similar. Find the missing dimensions, then find the area of both rectangles.



ANSWERS TO TRY IT

1. a) Rectangle, parallelogram, quadrilateral
   b) Rhombus, parallelogram, quadrilateral
   c) Parallelogram, quadrilateral
   d) Square, rhombus, parallelogram, rectangle, quadrilateral

2. a) Neither b) Trapezoid c) Isosceles Trapezoid

3. a) Perimeter = 22 units Area = 28 units^2
   b) Perimeter = 36 units Area = 65 units^2
   c) Perimeter = 24 units Area = 36 units^2
   d) Perimeter = 16 ft. Area = 7 ft.^2

4. a) Perimeter = 16 units Area = 12 units^2
   b) Perimeter = 12 units Area = 9 units^2

5. Mariana's room is 11 ft. by 11 ft.

6. Let x = the width = 300 m
   $2x$ = the length = 600 m

$$
\begin{aligned}
2(x) + 2(2x) &= 1800 \\
2x + 4x &= 1800 \\
\frac{6x}{6} &= \frac{1800}{6} \\
x &= 300
\end{aligned}
$$

7. a) Not similar b) Similar

8. a) $x = 12$ b) $z = 13.5$ m

End of Lesson 18